Quantity Theory of Money (Allais' viewpoint) The Case Study of Iran Economy

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Abstract

The relationship between price level and money as well as the effect of money on real variables has always been one of the important economic issues in the field of Monetary Economics. In this regard, we can cognize the Quantity Theory of Money as the most famous theory. This paper analyze Allais' model in the context of Quantity Theory of Money by using mathematical approach. The results show that economic and price growth have positive effect on income velocity of money and negative effect on relative desired money balances, as well as pure rate of interest –rate of time preference- has positive effect on desired money balances and versus negative effect on income velocity of money. So, the instability of income velocity of money leads to instability of money change effectiveness on price changes.

Keywords: Quantity Theory of Money, Velocity of Money, Pure Rate of Interest, Psychological Rate of Expansion

JEL: E31, E41, E51, E52

1. Introduction

Since Monetary sector is one of the important sectors in economy, the monetary policy analysis and its relationship with the price level are experimentally considered by economists and policymakers. Accordingly, how to set the monetary policies and to consider the relationship between these policies and macroeconomic variables are very important issues in every economy. However, there is a deep quarrel between economic schools about method of usage and its effectiveness on real variables. Indeed, throughout the history of economics, many economists have researched and presented theories about the meaning and concept of money, its duties, importance, types, determining factors of demand, and its effects on economic variables. Undoubtedly, the Quantity Theory of Money is the most famous theory in this field which has been presented by some scholars during many years; In fact, they have attempted to identify characteristics which can explain equality of a flow of money payments and a flow of commodity transactions.

The Quantity Theory of Money was the dominant theory in macroeconomics before 1930s. However viewpoints about the relations between macroeconomic variables as well as effects of money on these variables were changed by Keynes’s Revolution. Finally, the Theory was revived again with the formation of Chicago School in 1960s. The main point of new Quantity Theory is that there is a stable functional relation between real balances demand and a definite number of its determinants. However the relationship between money quantity and price level could be known as the greatest and the most important implication of the Quantity Theory of Money.
Respectively, proportion and type of this relation, its constancy or variability, and affecting factors have been surveyed and stated by economists. Likewise, Maurice Allais presented another version of Quantity Theory based on an especially psychological monetary theory. The reason for concentrating on Allais' model in this paper is that his thoughts in comparison with others such as Milton Friedman are really strange all over the world including Iran. Also, his analytical approach in explaining Quantity Theory of Money and in acquiring a demand function for money which can correctly interpret tentative observations and monetary behavior of economic agents concludes some different results and outstanding implications.

2. Literature

The start point of Quantity Theory of Money could be sought in Jean Bodin’s book¹: Reply to “De Malestroit”’s Wonderful considerations. He noted that price rising in France which was caused by enhance of foreign gold. This standpoint about the cause of price rising was stated by some other scholars in the middle of 17th century, such as Briscue (1694) and Hume (1752). Hume has clearly noted this view in his important paper, “About Money”. However the orthodox Quantity Theory of Money has been formed at classic economists' period and has been reconsidered subsequently during the history of economics.

2.1. The Classic Quantity Theory of Money

The Classic Quantity Theory of Money is expressed by Simon Newcomb and Irving Fisher’s “Equation of Exchange” (1911) with the best manner:

\[ MV = PQ \]

Where M is money, V is velocity, P the price level and Q is the total product². Equation (1) is basically an identity which simply implies that the product of the money quantity and the velocity or total expenditure must be equal to the product of the P and the Q or value of total output. Behind the restatement of the old Quantity Theory by Newcomb-Fisher, then, we have three pillars: firstly, that V and T are fixed with respect to the money supply. Secondly, that the supply of money is exogenous. Thirdly, the direction of causation runs from left (MV) to right (PT).

In the classical view, money is supposed as a Medium of Exchange which provides the possibility of obtaining commodities for people; thus, it has no utility itself. In the other word, money is not a good and hence the demand for money is neglected in the classical viewpoint. The Cambridge approach economists have varied view concerning the above mentioned topic.

2.2. The Cambridge Approach

Simon Newcomb's and Irving Fisher's Quantity Theory, relies entirely on the idea of a stable transactions demand for money. This requires that money is desired only for its medium of exchange function. An alteration on this point was brought in by several Cambridge economists in the earlier part of 20th century. In particular, A. C. Pigou (1917), Alfred Marshall (1923), D.H. Robertson (1922), John Maynard Keynes (1923), R.G. Hawtrey and Frederick Lavington (1921, 1922). These were the joint creators of what has since become known as the "Cambridge cash-balance" approach. In this approach, money has two functions: Medium of Exchange and Store of Value.

Hence, the proposition they advance is that money is desired as a store of value. The Cambridge story, then, is fundamentally different from the Fisher story. In Fisher, money is desired by agents in some fixed amount solely because it happens to be the medium of exchange. As Fisher noted, money yields no gains to the holder. However, in the Cambridge story, this is not the case. Money does increase utility in a way: namely, by enabling the divorce of sale and purchase as well as a hedge against uncertainty.

The Cambridge approach is that the sale and purchase of commodities are not constant and permanent; thus there is a need for a "temporary abode" of purchasing power, i.e. some temporary store of wealth. In particular, A.C. Pigou (1917) also allowed for money demand to involve a precautionary motive - with money holdings acting as a hedge against uncertain situations. As it is in its store-of-wealth and precautionary modes that money yields utility to the consumer, then it is demanded for itself in a way.

¹. 1568
². It has been used of T_level of transactions_ instead of Q In the primary writings.
How much of it is demanded depends partly on income and partly on other items, notably wealth and interest rates. The first part is obviously implied in transactions terms: the higher the volume of income, the greater the volume of purchases and sales, hence the greater the need for money as a temporary abode to overcome transactions costs. Thus, Cambridge theorists regarded real money demand as a function of real income, i.e.

\[ \frac{M}{P} = kY \quad (2) \]

Where \( k \) is the famous "Cambridge constant", and is an indication for a part of income which is kept as cash. However, this is really misleading for the "constant" \( k \) is not constant at all. Rather, it relies on other components, such as interest (the opportunity cost of money) and wealth. The main points of the Cambridge approach were two: (1) neutrality of money remains; (2) money yields services and is demanded by choice.

### 2.3. The Neo-Classical Theory

This approach formed in 90s in 19\(^{th}\) century and continued its existence by 20\(^{th}\) century in 30s. The Neo-Classical economic viewpoint is based on Benefit or Utility and formed into the microeconomic framework. Its monetary theory formed on two pillars: Irving Fisher’s transaction velocity; Marshall’s and Walras’ cash balances approach. The Neo-Classical Quantity Theory of Money could be written as follow:

\[ M \cdot V + M' \cdot V' = P \cdot T \quad (3) \]

Where \( M \) is money (currency and coinage), \( V \) is velocity circulation of this money, \( M' \) deposits, \( V' \) velocity with response to \( M' \), \( P \) is price level, and \( T \) is total transactions.

### 2.4. Monetarism

"Monetarism" began with Milton Friedman's article entitled "The Quantity Theory of Money: A restatement" (1956) which was followed up later in Friedman (1968, 1969, 1970, 1971). Milton Friedman proposed a money demand function in the following general form:

\[ M^d = f(Y_p, r_b, r_e, r_m, \pi^e) \]

where money demand is positively related to permanent income \( Y_p \), negatively related to expected interest rates on bonds \( r_b \), the expected rate of return on equity \( r_e \) and expected inflation, \( \pi^e \). "Permanent income", is the expected average long-run income.

Friedman claims that velocity of circulation is constant in long-run, based on his observations and considerations of monetary History of United States. Then he proposed following relation:

\[ \frac{gM}{gy} = \frac{gv}{gY} \]

\( Y \) grows with a stable rate, thus an increase in M leads to an increase in P or Y or both of them.

One of the political implications of Friedman’s Restatement for policymakers is that M must grow with the rate which in Y grows, if they want to control prices constantly.

### 2.5. M. Allais: “A Restatement of the Quantity Theory of Money”

Maurice Allais expressed, in an article entitled “A Restatement of the Quantity Theory of Money” (1965), another version of Quantity Theory based on the monetary theories surveyed in his priori papers: he wrote “Hereditary Effects and the Relativity of Time in the Social Sciences”(1963) and introduced some psychological postulates into the monetary theory. Then he wrote another paper, “The Hereditary, Relativistic and Logistic Formulation of the Demand of Money” (1964), and formulated a function for money demand. Finally, he presented a different statement of the Quantity Theory following to above papers.

Allais formulated the postulates based on the mentioned principles and did his analyses based on this formulation:

\[
\begin{align*}
(1.1) & \quad \frac{M}{P} = \phi(u) = \phi_0 e^{-Ku} \\
(1.2) & \quad \frac{du}{dt} = \xi(x - u) \\
(1.3) & \quad x = \frac{1}{P} \frac{dP}{dt} \\
(b) & \quad \frac{M}{D} = \phi(u) = \phi_0 (1 - Ku) \quad \text{(Allais 1954)} \\
(b) & \quad u = \int_{-\infty}^{x} \xi(\theta)e^{-\tau(\theta)}d\theta \\
(b) & \quad \int_{-\infty}^{x} e^{-\tau(\theta)}d\theta \\
(b) & \quad x = \frac{1dD}{Dt} = \frac{1dP}{Pdt} + \frac{1dQ}{Qdt} \quad \text{(Allais)}
\end{align*}
\]
Where D is total expenditure, P is prices level, Q the economic activity index, and M is money. Equation (1.1) indicates the demand for real balances. As it could be seen, money demand is a diminishing function of weighted average of earlier total outlay growth rates u. Allais says: “the monetary behavior of economic agents is a function of a psychological rate of expansion. This rate is found as the weighted average of the total nominal expenditures growth rates.” Thus, the relation (1.2) is the weighted average of the total nominal expenditures growth rates.

Based on the Allais’ definition, it assumes that the relative desired money balances is a function of psychological rate of expansion z:

$$
\Phi_D = \Phi(z) = \frac{M_D}{D}
$$

(4)

According to definition of velocity circulation of money V, it could be written as the inverse of the relative desired money balances:

$$
V_D = \frac{D}{M_D} = \frac{1}{\Phi_D}
$$

(5)

Finally, Allais concludes that, in a given time, there is a proportional relation between price level and money quantity; But the coefficient of this proportionality is not constant. Also, circulation velocity of money appears as a function of psychological rate of expansion.

3. Theoretical Model

This section has been specified as the model of this research, so that it could be estimated. The specification of the model has two advantages:

1) One can determine the optimal quantity of variables, based on the model specified.
2) It provides the circumstances in which the calculated values of variables could be compared to the observed values of variables.

According to the above passage, the model is specified firstly, and then the acquired results of the model and its implications are interpreted. The calculation of the variables and also its comparison with the observed values in Iran is surveyed in section 4.

What has distinguished Allais’ model from the other studies in the context of Quantity Theory is identifying the formulation of monetary behavior of economic agents based on their expectations. Indeed, Allais tries to explain how the change in the key variables_ such as national income_ leads to change in economic agents’ expectations. He also expresses how the change in the expectations induces in the behavior of agents. In other word, based on Allais’ explaining, change in national income in over time is the determinant factor of monetary decisions of economic agents.

The index which shows the effect of national income changes on the expectations of economic agents is called psychological rate of expansion. This rate could be known as the weighted average of total nominal expenditures growth rates.

$$
Z' = \int_{-\infty}^{\tau} x'(T')e^{-F'(t'-T')}dT'
$$

$$
\int_{-\infty}^{\tau} e^{-F'(t'-T')}dT'
$$

(6)

The equation (6) shows that the inherited nature of the relation between z psychological rate of expansion, x are growth rates of nominal outlays, F is coefficient of forgetfulness which specifies the effect of preceding variables values on the expectations of agents. The weighting coefficients $e^{-F'(t'-T')}$ decline exponentially with time. The relation (6) specifies the hereditary nature of the link that exists between the psychological rate z and the growth rates x.

The forgetfulness function has a diminishing form, over time:

3. M. Allais, (1965)
Indeed, the closer events (t) or the recent events have more effectiveness on expectations rather than earlier events (T), with respect to the time period considered. In other word, the forgetfulness respected to the former is smaller rather than later. Hence, economic agents forget the earlier events more than closer events.

According to the above explanation, the desired money balances can be written as a function of psychological expansion rate:

$$\phi_D = \frac{M_D}{D},$$  \hspace{1cm} (8)

$$\phi_D = \phi(z).$$  \hspace{1cm} (9)

The relations (6), (7) specify that monetary behavior of economic agents is a function of a psychological expansion rate, and this rate by itself could be known as the weighted average of total nominal outlay growth rates, and also, the weighting coefficients of outlay growth rates decline over time; thus, any backward to the earlier times of period considered gives us the smaller weighting coefficients.

The value of $\phi_0$ is defined by relation:

$$\phi_0 = \phi_D(0)$$  \hspace{1cm} (10)

$\phi_0$ is not the initial value of $\phi$, but the value of $\phi$ for $z=0$. It follows from assumption that in a stationary process in which total outlay $D$ is constant, the growth rates of $D$ is equal to zero $x(t)=0$, so that $z(t)=0$. It means that in a stationary process, all of economic agents have a same behavior. Thus, the forgetfulness rate of all of them is equal to its stationary value.

As we know, the circulation velocity of money is defined by relation:

$$D = M_D V_D$$  \hspace{1cm} (11)

where $V_D$ is circulation velocity of money, $M_D$ is money demand, and $D$ is total nominal expenditures.

Total nominal expenditures can be calculated as follow:

$$D(t) = \sum_i p_i(t)q_i(t) = P(t)Q(t)$$  \hspace{1cm} (12)

Where $p_i$ and $q_i$ represent the price and volume of the transaction $I$, and $P$ and $Q$ are indices of price and of economic activity. Hence, the growth rate of total nominal expenditures can be written as follow:

$$x(t) = \frac{1}{D(t)} \frac{dD(t)}{dt} = \frac{d}{dt} \log D = \frac{\dot{D}}{D}$$  \hspace{1cm} (13)

The equation (11) can be written as:

$$V_D = \frac{D}{M_D}$$

And also the equation (8) has been represented as follow:

$$\phi_D = \frac{M_D}{D}$$  \hspace{1cm} (14)
Thus circulation velocity of money appears as the inverse of desired money balances. If we identify the relative desired money balances function as follow:

$$\frac{\phi_D}{\phi_0} = \psi(Z) \quad (15)$$

Where relative desired money balances $$\psi(z)$$ could be calculated by the following relation:

$$\psi(Z) = \frac{1 + b}{1 + be^{az}} \quad (16)$$

Hence, with regard to relations (14) and (15):

$$V \approx \frac{1}{\phi_0\psi(Z)} \quad (17)$$

Relation (17) implies that the circulation velocity of money is a function of psychological expansion rate. Finally, we can write the relation (8) in following form:

$$\phi_D = \frac{M_D}{D} = \frac{M_D}{PQ}$$

Then

$$M_D = PQ\phi_D \quad (18)$$

This above-mentioned relation specifies that at any given moment, demand for money ($$M_D$$) is proportional to the multiplying of the price level and the economic activity level. On the other hand This is the foundation of the Cambridge version of the Quantity Theory, in which assume that peoples always want to hold a proportion of own income at the money form.

According to relations (15) and (18), Demand for money ($$M_D$$) may be written as below:

$$M_D = \phi_0\psi(Z)D \quad (19)$$

4. The Model Calibration

In this section, according to the relations acquired and gained by preceding section, it is attempted the values of various variables are calculated, and then are compared with their empirical values in Iranian economy.

4.1. Constant Parameters of the Model

To calculate the values of variables noted in section 3, it should firstly determine the constant parameters of the model_ a, b, and F (Z=0).

In order to simplify and based on Allais’ calculations, the values of a and b are assumed to be equal to one.

a=1 \quad (20)

b=1 \quad (21)

As noted in section 3, relative desired money balances $$\psi(z)$$ can be calculated by following relation:

$$\psi(Z) = \frac{1 + b}{1 + be^{az}} \quad (22)$$

Thus, according to (20), (21), (22):

$$\psi(z) = \frac{2}{1 + e^z} \quad (23)$$
Thus, relative desired money balances is a diminishing function of psychological expansion rate, and its maximum value is 2.

$$\psi(-\infty) = \psi_{\text{Max}} = 2$$
$$\psi(+\infty) = 0$$
$$\psi(Z = 0) = 1$$

There is an important assumption with respect to determining of forgetfulness parameter, which is very useful in empirical studies. This assumption says that economic agents’ imagination about the future, resemble to their view about the past events. In other words, economic agents discount the future with the rate in which they forget the past; or they forget the past with the future discounting rate. As Allais says, “economic agents are assumed to take the past into consideration as they do the future.” Thus:

$$F = i$$

(24)

Where i is pure rate of interest, or time preference rate. Therefore it could be written for Z=0 that:

$$F_0 = i_0$$

(25)

The important note here is that the pure rate of interest is time preference rate.

4.2. Calculated and observed values

In this section, we compare the calculated with the observed values of money quantity and of relative desired money balances.

To utilize the relation (6) for empirical studies, it must be written in the discrete form:

$$z = \frac{x_t + kx_{t-1} + \ldots + k^px_{t-p} + \ldots}{1 + k + \ldots + k^p + \ldots}$$

(26)

$$x_t = \frac{d\ln D}{dt}$$

(27)

$$k = e^{-pF_0}$$

(28)
Where \( x_t \) is growth rate of total nominal outlay, \( k \) is weighting coefficient of \( x_t \), and the exponent of \( k \) – in this relation, \( p \) implies the time interval. As implied in section 3, weighting coefficients decline when time interval increases. Therefore:

\[
0 < k \leq 1
\]

Desired money balances \( M_D \) are a psychological concept and of course no statistical data to measure it are available. However, at any given moment, economic agents are in a position to adjust their money balances \( M \) from existing towards desired levels \( M_D \) either by spending more or buying less. Naturally, this adjustment is never perfect, but it can reasonably be suggested that the discrepancy between the actual and the desired value of money holdings is always relatively small. Thus:

\[
\left| \frac{M - M_D}{M_D} \right| < \xi
\]

(29)

Where \( \xi \) is a small quantity.

It further follows that it is possible to write:

\[
M_D \approx M
\]

(30)

\[
\phi_D = \frac{M_D}{D}
\]

\[
\phi = \frac{M}{D}
\]

\[
\phi_D \approx \phi
\]

(31)

Now, the calculated value of money \( M^* \) could be obtained by following relation:

\[
M^* = D\phi_D
\]

(32)

Thus, from the relations (19) and (32):

\[
M^* = \phi_0 D \psi^*(z)
\]

(33)

Where \( \psi^*(z) \) is calculated value of \( \psi \) which is obtained from the relation (23).

Finally, from the relations (23) and (33), we can write:

\[
M^* = \frac{2\phi_0 \psi^*}{1 + e^z} D
\]

(34)

The scheme (4.1) shows the calculated money quantity \( (M^*) \) and the calculated desired money balances \( (\psi^*(z)) \) with respect to the various forgetfulness rate _time preference rate_ values. As the scheme shows, with regard to the theoretical explanation, desired money balances is a diminishing function.

Scheme (4.1): the calculated money quantity and the calculated desired money balances with respect to the forgetfulness rates 5%, 10%, 15%
(Billion Rials)

<table>
<thead>
<tr>
<th>Year</th>
<th>$F=5%$</th>
<th>$F=10%$</th>
<th>$F=15%$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$\psi$</td>
<td>$M$</td>
</tr>
<tr>
<td>1986</td>
<td>3081.423</td>
<td>0.8438</td>
<td>3365.536</td>
</tr>
<tr>
<td>1987</td>
<td>753.2337</td>
<td>0.1694</td>
<td>2030.263</td>
</tr>
<tr>
<td>1988</td>
<td>849.487</td>
<td>0.171</td>
<td>2305.041</td>
</tr>
<tr>
<td>1989</td>
<td>556.524</td>
<td>0.09</td>
<td>2150.656</td>
</tr>
<tr>
<td>1990</td>
<td>296.646</td>
<td>0.035</td>
<td>1883.278</td>
</tr>
<tr>
<td>1991</td>
<td>191.1206</td>
<td>0.016</td>
<td>1813.257</td>
</tr>
<tr>
<td>1992</td>
<td>191.3904</td>
<td>0.012</td>
<td>2086.155</td>
</tr>
<tr>
<td>1993</td>
<td>136.8643</td>
<td>0.0057</td>
<td>2110.592</td>
</tr>
<tr>
<td>1994</td>
<td>172.3445</td>
<td>0.0055</td>
<td>2779.446</td>
</tr>
<tr>
<td>1995</td>
<td>178.4909</td>
<td>0.004</td>
<td>3328.855</td>
</tr>
<tr>
<td>1996</td>
<td>226.4934</td>
<td>0.0038</td>
<td>4404.7</td>
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<tr>
<td>1997</td>
<td>351.2136</td>
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<td>1998</td>
<td>568.7436</td>
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<td>2002</td>
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<td>2003</td>
<td>1939.547</td>
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<td>2004</td>
<td>2467.902</td>
<td>0.007</td>
<td>39274.89</td>
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<tr>
<td>2005</td>
<td>3253.165</td>
<td>0.0074</td>
<td>50907.63</td>
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<tr>
<td>2006</td>
<td>4377.015</td>
<td>0.0082</td>
<td>65975.49</td>
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<td>2007</td>
<td>5461.4708</td>
<td>0.0079</td>
<td>83907.65</td>
</tr>
<tr>
<td>2008</td>
<td>5812.251</td>
<td>0.0081</td>
<td>87432.5</td>
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<tr>
<td>2009</td>
<td>6132.521</td>
<td>0.0075</td>
<td>88261.41</td>
</tr>
<tr>
<td>2010</td>
<td>6684.325</td>
<td>0.0062</td>
<td>91352.62</td>
</tr>
<tr>
<td>2011</td>
<td>6979.231</td>
<td>0.0058</td>
<td>95624.53</td>
</tr>
<tr>
<td>2012</td>
<td>7351.523</td>
<td>0.0043</td>
<td>99842.35</td>
</tr>
</tbody>
</table>

Now, we can compare the various values of $\psi^*(z)$ with the observed values of $\psi$ in economy of Iran in this section. First notable point about scheme (4.1) is that all values of $\psi^*(z)$ are diminishing. Second point is that the values of $\psi^*(z)$ respected to the higher time preference rate have a lower deviation from the observed $\psi$. Thus, it seems that time preference rate have a high value in Iran. In other words, if we note the values of time preference rate in [5% , 15%] for economy of Iran, it is closer to 15% rather than to 5%. This is also visible in desired money balances curves. Figure (4.2) shows the calculated $\psi$ respected to forgetfulness rate 5%_ figure (4.2.A) _, 10%_figure (4.2.B)_ , and 15% _figure (4.2.C) _ in contrast to the observed $\psi$ _ Figure (4.2.D) _.


Figure (4.2.A): desired money balances respected to forgetfulness rate 5%

Figure (4.2.B): desired money balances respected to forgetfulness rate 10%

Figure (4.2.C): desired money balances respected to forgetfulness rate 15%
As it is mentioned earlier, and according to the theoretical explanation, the calculated and observed values of desired money balances are diminishing functions of psychological expansion rate $Z$, while desired money balances have been drawn with respect to the time period in this section. Because the psychological expansion rate has an increasing trend, in the time period considered, we can use trend term for drawing of $\psi$ curves.

As it is noted, the deviation of $\psi^*(z)$ curve respected to the forgetfulness rate 15% from the observed $\psi$ curve is lower than the deviation of $\psi^*(z)$ curve respected to the forgetfulness rates 5% and 10%. In other words, the $\psi^*(z)$ respected to the forgetfulness rate 15% has gained a better estimation of desired money balances in Iran.

We can also compare the calculated ($M^*$) and the observed ($M$) values of money quantity. It could also be seen that the values of $M^*$ respected to the higher forgetfulness rates have a lower deviation from the observed $M$. Figure (4.3) shows the various values of $M^*$ with respect to the forgetfulness rates 5%, 10%, and 15% versus the observed values of $M$.

**Figure (4.3.A): Money quantity respected to the forgetfulness rate 5%**
Figure (4.3.B): Money quantity respected to the forgetfulness rate 10%

Figure (4.3.C): Money quantity respected to the forgetfulness rate 15%

Figure (4.3.D): Observed quantity of money

Gross national expenditures and its progressive growth could be seen in figure (4.4).

Figure (4.4): Gross national expenditures
Figure (4.5) shows that the proportion of money quantity to GNP, Real quantity of money, is declined over the time period considered. Therefore, the money quantity couldn’t support all marginal transactions.

![Figure (4.5): Real quantity of money](image_url)

### 4.3. A Measure for the Goodness of the Fit

Now, a measure for the goodness of the fit must be identified. It can be the minimum value of:

$$e^2 = \frac{1}{N} \sum_{n=1}^{n=N} \left[ L_n M_n - L_n M^* \right]^2$$  \hspace{1cm} (35)

In fact, if the sum of squares of the deviations of the calculated from the observed values of the natural logarithm of M is minimum, then it is considered as the criterion of the best possible fit. The scheme (4.2) shows the values of $e^2$ respected to the various forgetfulness rates.

<table>
<thead>
<tr>
<th>Scheme (4.2): Values of $e^2$ respected to the forgetfulness rates 5%, 10%, 15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>F=5%</td>
</tr>
<tr>
<td>$e^2$</td>
</tr>
</tbody>
</table>

As it could be seen from the scheme (4.2), the calculated M with respect to the F=15% has the minimum deviation from the observed M. In fact, F=15% has a better statement of the economic circumstances of Iran.

### 5. Conclusions

According to relation $\phi_0 = \phi_D (z=0)$, we can say that $\phi_0$ is value of $\phi_D$ in stationary process. In fact the proportion of $\frac{M_D}{D}$ (Cambridge coefficient) only in the stationary process is constant. Otherwise, $\phi_D$ could be calculated from the relation $\phi_D = \phi_0 \psi(Z)$. This implies an important and noticeable point: the Cambridge Coefficient is not constant. In order to clarify the topic, we must explain the classical assumptions in which the Cambridge economists thought and worked. The full employment assumption and its output, means that total product (or output) is constant over the time and thus the growth rate of output is zero. In other words, existence of stationary condition in an economy is the one of the implications of the full employment assumption in classical viewpoint.

According to the relation (6), zero growth rate for output leads to $Z=0$ and then $\phi_0$ is $\phi_D$ with respect to $Z=0$. Thus $\phi_0$ exactly is the Cambridge Coefficient which is corresponding with the stationary process in economy.

On the other hand, omission of the full employment assumption (stationary process) concludes that the proportion $\frac{M_D}{D}$ is not constant and is a function of psychological rate of expansion or weighted average of growth rates of past nominal outlays.

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4. The growth rate of output in the stationary process is zero.
Based on relations (14) and (17), circulation velocity of money is equivalence with inverse of $D$. Thus, according to above paragraph, circulation velocity of money is also constant only in stationary process; otherwise circulation velocity of money is not constant. In other words, omission of the full employment assumption (stationary process) concludes that circulation velocity of money is a function of psychological expansion rate.

Relative desired money balances $\psi (Z)$ is a diminishing function of psychological expansion rate, or weighted average of growth rates of past nominal outlays. Since that nominal outlays are equal to the product of output and price level, and then nominal outlay growth is equal to the sum of output growth and price growths. In viewpoint of Cambridge school, the higher income leads to the more transactions, and this needs the more money as the store of value. Thus the higher income the more demand for money ($M_p$). But the point is that income (or outlay) growth leads to diminish relative desired money balances $\psi (Z)$. The higher growth of income (or outlay) the less desired money balances. Also the higher growth of prices the less desired money balances. In classical viewpoint the demand for money is a diminishing function from interest rate; because in this view, money has an equilibrium price and quantity which refers the supply and demand balance, like to every commodity. So it could be said that interest rate is the price of money and thus its equilibrium quantity could be determined in the market.

But based on the conclusion of this study, there are three factors which determine relative desired money balances: output growth, price growths, and time preference rate. Both output growth and price growths have a negative effect on relative desired money balances; while Time preference rate (pure rate of interest) affect on relative desired money balances positively. In fact, the higher time preference, the lower psychological expansion rate, and thus the greater relative desired money balance.

According to relation (17) output growth and price growths have a positive effect on circulation velocity of money; the greater output growth_ or price growths_ the higher circulation velocity of money; because the quantity of money must covers whole of marginal transactions. Also it's concluded that the time preference rate has a negative effect on circulation velocity of money.

A notable point in this regard is that the higher time preference rate leads to the higher prices. In other words, the higher time preference rate causes the higher proportion of consumption to income in the society, and then price level will increase. So we can say that the higher time preference rate leads to the greater relative desired money balances, the lower circulation velocity of money, and the higher price level. Finally we must imply that increase in the price level impacts two effects: it decreases the desired money balances; on the other hand, it leads to the higher circulation velocity of money. Thus, desired money balances is a very little quantity, and circulation velocity of money is a very high quantity, in the hyperinflations.

References

Appendix

Scheme (A.1): the calculated money quantity and the calculated desired money balances with respect to the forgetfulness rates 20%, 25%, 30%
(Billion Rials)

<table>
<thead>
<tr>
<th>Year</th>
<th>F=20% M</th>
<th>F=20% Ψ</th>
<th>F=25% M</th>
<th>F=25% Ψ</th>
<th>F=30% M</th>
<th>F=30% Ψ</th>
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<tr>
<td>1986</td>
<td>3511.244</td>
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<td>3545.206272</td>
<td>0.9708</td>
<td>3562.735104</td>
<td>0.9756</td>
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<td>1987</td>
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<td>3342.863664</td>
<td>0.7518</td>
<td>3514.94244</td>
<td>0.7905</td>
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<td>1988</td>
<td>3573.707</td>
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<td>3777.484704</td>
<td>0.7604</td>
<td>3974.208</td>
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<td>1989</td>
<td>2909.384</td>
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<td>4220.307</td>
<td>0.6825</td>
<td>4496.71392</td>
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<td>1990</td>
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<td>4842.95784</td>
<td>0.5714</td>
<td>5297.25</td>
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<td>1991</td>
<td>3057.93</td>
<td>0.256</td>
<td>5884.126704</td>
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<td>6580.522536</td>
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<td>1992</td>
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<td>7504.0986</td>
<td>0.4705</td>
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<td>1993</td>
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<td>9508.46688</td>
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<td>11141.23392</td>
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<td>12120.52</td>
<td>0.3868</td>
<td>12919.56893</td>
<td>0.4123</td>
<td>15100.50998</td>
<td>0.4819</td>
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<td>1995</td>
<td>12025.82</td>
<td>0.2695</td>
<td>17260.0681</td>
<td>0.3868</td>
<td>20374.73395</td>
<td>0.4566</td>
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<td>1996</td>
<td>21594.36</td>
<td>0.3623</td>
<td>23418.22301</td>
<td>0.3929</td>
<td>27846.74354</td>
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<td>1997</td>
<td>38380.62</td>
<td>0.5464</td>
<td>31567.07837</td>
<td>0.4494</td>
<td>36968.74354</td>
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<td>1998</td>
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<td>40404.48984</td>
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<td>1999</td>
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<td>51619.554</td>
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<td>59877.006</td>
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<td>2000</td>
<td>55613.96</td>
<td>0.3992</td>
<td>66020.67713</td>
<td>0.4739</td>
<td>76747.81817</td>
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<td>2001</td>
<td>87379.42</td>
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<td>84396.91104</td>
<td>0.5235</td>
<td>96826.71398</td>
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<td>2002</td>
<td>86741.32</td>
<td>0.3898</td>
<td>106782.4091</td>
<td>0.4819</td>
<td>123778.0737</td>
<td>0.5586</td>
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<td>2003</td>
<td>131643.4</td>
<td>0.4819</td>
<td>136478.5697</td>
<td>0.4996</td>
<td>156994.063</td>
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<td>2004</td>
<td>159496.9</td>
<td>0.4524</td>
<td>174938.962</td>
<td>0.4962</td>
<td>201451.2755</td>
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<td>2005</td>
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<td>223149.5283</td>
<td>0.5076</td>
<td>256296.641</td>
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<td>2006</td>
<td>260966.2</td>
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<td>280929.635</td>
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<td>2007</td>
<td>265014.5</td>
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<td>298654.84</td>
<td>0.394955</td>
<td>396542.481</td>
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<td>2008</td>
<td>282315.4</td>
<td>0.399668</td>
<td>342241.741</td>
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<td>445268.954</td>
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<td>2009</td>
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<td>0.39509</td>
<td>369847.357</td>
<td>0.369358</td>
<td>495865.254</td>
<td>0.465566</td>
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<td>2010</td>
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<td>0.390512</td>
<td>397856.241</td>
<td>0.35656</td>
<td>556342.741</td>
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<td>2011</td>
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<td>0.343761</td>
<td>61875.954</td>
<td>0.444838</td>
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<td>2012</td>
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<td>495624.852</td>
<td>0.330963</td>
<td>679854.542</td>
<td>0.434474</td>
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</table>
Figure (A.2): Money quantity respected to the forgetfulness rate 25%

Figure (A.3): Money quantity respected to the forgetfulness rate 30%

Figure (A.4): desired money balances respected to forgetfulness rate 20%
Figure (A.5): desired money balances respected to forgetfulness rate 25%

Figure (A.6): desired money balances respected to forgetfulness rate 30%

Figure (A.7): Observed quantity of money

Figure (A.8): observed desired money balances